

THE TERM STRUCTURE OF INTEREST RATES

Term structure basics

The yield curve - the level of rates and the term spread

The "normal" pattern and other possible shapes/movements

Three views on the term structure relationship

pure expectations

risk or liquidity premium

market segmentation

Examine and evaluate each view

Interpreting yield-to-maturity when there is a sloped yield curve

First view: Pure Expectations Theory

Assume (i) risk neutral investors, (ii) investors can freely switch between different maturities, (iii) default free bonds, and (iv) pure discount bonds.

At any point in time investors have **three** maturity choices

(i) buy bonds with maturity equal to investment horizon

e.g., consider a three year investment horizon --- investor buys three-year bonds

$$\text{return} = (1 + y_3)^3$$

y_n = a nominal interest rate maturing at time "n"

(ii) buy short-term bonds and roll over

e.g., buy sequence of three one-year bonds

$$\text{expected return} = (1 + y_1) * (1 + {}_1f_1) * (1 + {}_2f_1)$$

${}_mf_n$ = a nominal interest rate on a bond that begins at $t=m$ and matures n periods later

e.g., buy a two-year bond followed by a one-year bond

$$\text{expected return} = (1 + y_2)^2 (1 + {}_2f_1)$$

(iii) buy a longer term bond and sell before it matures

e.g., buy a five-year bond and sell after three years

$$\text{expected return} = \left[\frac{(1 + y_5)^5}{(1 + {}_3f_2)^2} \right]$$

What must be the relationship between these expected returns?

They must all be equal.

$$(1 + y_3)^3 = (1 + y_2)^2 (1 + {}_2f_1)$$

$${}_2f_1 = \frac{(1 + y_3)^3}{(1 + y_2)^2} - 1$$

and

$$(1 + y_3)^3 = \left[\frac{(1 + y_5)^5}{(1 + {}_3f_2)^2} \right]$$

$${}_3f_2 = \left[\frac{(1 + y_5)^5}{(1 + y_3)^3} \right]^{1/2} - 1$$

Interpretation: Long-term rates are the geometric average of the current and expected future short-term interest rates.

Implication --- market forecasts can be derived from long- and short-term rates

$${}_m f_n = E({}_m y_n)$$

Alternative interpretation: The expected holding period return over the period t to $t+1$ is the same for every bond regardless of maturity.

A Second View: the Liquidity or Risk Premium Theory

A different interpretation of ${}_m f_n$: the expected future short-term rate plus a premium

$${}_m f_n = E({}_m y_n) + {}_m L_n$$

What does the premium compensate investors for?

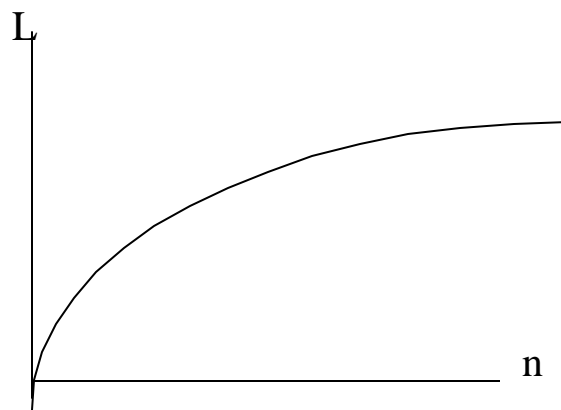
less liquidity for long-term bonds

bearing risk - risk minimizing position is maturity equal to time horizon. Risk arises when investor has to depart from preferred maturity habitat. Risk premium is inducement to depart from maturity preferences (*preferred maturity habitat*). [Remove assumption of risk neutrality.]

Mismatch #1. Issuers prefer long-term, investors prefer short-term: investors must be compensated for bearing price risk.

Mismatch #2. Issuers prefer short-term, investors prefer long-term:
investors must be compensated for bearing reinvestment risk.

Typical view of ${}_mL_n$ embracing Mismatch #1 - given starting point “m”



Third View: Market Segmentation Theory

Emphasizes the role of maturity preferences

Strongest form - institutional and legal constraints force binding maturity preferences. Result supply and demand within each maturity class determine rates. No movement between maturity classes.

Weaker forms - reduces to a version of risk premium theory. Compensation required to induce participants to depart from preferences. Investors will move in a continuum of preferred maturities.

Policy question that arises periodically: Can the Treasury influence interest rates and economic activity by varying maturity composition of federal debt?

Clinton administration proposal - reduce maturity of outstanding Treasury debt

- ◆ to lower interest rates for financing private economic activity
- ◆ to lower interest cost of federal debt when yield curve is steeply upward sloping

Empirically, what can we say about these 3 views?

1. Observe that most participants have distinct maturity preferences, but participants can and do switch within their preference region.
2. Some participants switch all across the maturity spectrum (e.g., dealers and mutual fund managers).
3. Yield curve has some forecasting ability, given an interest rate regime.
4. Yield curve shifts and rotations give similar predictions to surveys of market participants regarding major shifts in rates. There is some evidence that the market outpredicts the experts.

5. Implied or forward rates frequently appear to be upward, biased estimates of future interest rates. Upward bias means positive term premiums. There is, however, some evidence that term premiums have been negative during certain periods.

6. Maturity composition of the supply and demand for bonds can change substantially across long time intervals, but under most conditions will not change much over short time intervals.

YIELD-TO-MATURITY WITH SLOPED YIELD CURVES

For a pure discount bond with three years to maturity and no risk of default, the yield-to-maturity is the solution for y , given P_0 from the market,

$$P_0 = \frac{M}{(1+y)^3}$$

where

$$y = y_3$$

y_3 = the appropriate interest rate for discounting a single cash flow from three periods from now back to today

For a coupon bond, with three years to maturity, the yield-to-maturity is the solution for given P_0 from the market,

$$P_0 = \frac{C}{(1+y)} + \frac{C}{(1+y)^2} + \frac{C+M}{(1+y)^3}$$

In pricing the bond, the market will use the appropriate interest rate to discount each cash flow or

$$P_0 = \frac{C}{(1+y_1)} + \frac{C}{(1+y_2)^2} + \frac{C+M}{(1+y_3)^3}$$

where y_1 is the interest rate that discounts a single cash flow one period from now back to today, etc.

If $y_1 < y_2 < y_3$, then $y < y_3$.

If $y_1 > y_2 > y_3$, then $y > y_3$.

The value of y that solves the pricing equation depends on the relative sizes of C , M , y_1 , y_2 , and y_3 .